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Wave number selection under the action of accelerated rotation

O Krivonosova¹, D Zhilenko¹ and M Gritsevich^{2,3}

¹ Moscow State University, GSP-1, Leninskie Gory, 119991, Moscow, Russia

² Department of Physics, University of Helsinki, Gustaf H  llstr  min katu 2a, FI-00014 Helsinki, Finland

³ Institute of Physics and Technology, Ural Federal University, Mira str. 19, 620002, Ekaterinburg, Russia

E-mail: olga@imec.msu.ru

Abstract. Unsteady viscous incompressible flows in a spherical layer due to an increase in the rotation velocity of the inner sphere with constant acceleration are investigated. The acceleration starts at the Reynolds numbers Re corresponding to a stationary flow and ends at Re higher than the stability limit of the stationary flow, whereupon the rotation velocity of the inner sphere remains constant. The outer sphere is fixed and the spherical layer thickness is equal to the inner sphere radius. The inner sphere acceleration effect is studied on both the formation of one of two possible secondary-flow structures after the acceleration has been stopped, namely, traveling azimuthal waves with wavenumbers of 3 or 4, and on the change in the flow structure during the action of the acceleration. It is shown that not only an increase in the acceleration but also a decrease in Re corresponding to the acceleration onset can lead to an increase in the deviation of the instantaneous velocity profiles from their stationary values and can be accompanied by a variation in the secondary flow wavenumber.

1. Introduction

After the loss of stability three-dimensional viscous incompressible flows can be nonunique, namely, several different flow patterns may exist at the same Reynolds numbers Re but different flow prehistory. In spherical layers it is the influence of a constant acceleration of the inner sphere on transition to different secondary flows that is most completely experimentally studied. Thus, at the same Reynolds numbers but different accelerations of the inner sphere secondary flows with different numbers of the Taylor rings can be formed in thin layers [1], while in wide layers azimuthal waves with different wave numbers can be generated [2]. In the case of thin layers the experimentally obtained results on the formation of different stationary patterns, symmetric about the axis of rotation, under the action of acceleration are confirmed by direct calculations [3]. In the case of wide layers the numerical results are restricted to the calculations of the structures of unsteady axisymmetric flows formed in the process of internal sphere acceleration [4] and to an analysis of their stability in the quasi-stationary approximation [5]. In the wide layer with relative gap size $\delta = (r_2 - r_1)/r_1 = 1$ under stationary boundary conditions the growth rate of mode 4 is higher than that of mode 3 over a wide Re range [5,6]; however, in stationary flows developing after a stepwise change in the inner sphere velocity the relation between the growth rates can be opposite [5].



At present, the available experimental results on the inner sphere acceleration effect on the secondary flow formation in a wide $\delta = 1$ layer have made it possible to establish the following patterns. Under the influence of the acceleration the mode realized in the secondary flow exceeds the noise level and starts to grow earlier than the other modes [7]. The same effect was previously numerically found in the case of axisymmetric secondary flows in the cylindrical Couette flow [8]. The existence of the critical acceleration is established, such that a four-vortex flow pattern is realized below it and a three-vortex pattern above it. For small values of Re_i , that is, the Reynolds number associated with the acceleration onset, its small variations lead to large variations in the critical acceleration, whereas at high Re_i the critical acceleration depends on it only slightly. It was found that there exist indefiniteness domains, that is, the Re_i ranges on which the four-vortex pattern is formed at small accelerations, while with increase in the acceleration the azimuthal wavenumber is chosen in a random fashion. It was shown that an increase in the acceleration is accompanied by an increase in the nonequilibrium of the azimuthal velocity profiles, that is, the deviation from the profiles corresponding to stationary Re values [9].

2. Formulation of the problem

Isothermal flow of viscous incompressible fluid is described by the Navier–Stokes and continuity equations:

$$\frac{\partial \mathbf{U}}{\partial t} = \mathbf{U} \times \text{rot} \mathbf{U} - \text{grad} \left(\frac{p}{\rho} + \frac{\mathbf{U}^2}{2} \right) - \nu \text{rot} \text{rot} \mathbf{U}, \quad \text{div} \mathbf{U} = 0.$$

In the case of a flow in a spherical layer it is natural to use a spherical coordinate system with the radial (r), polar (θ), and azimuthal (φ) directions, corresponding velocity \mathbf{U} components are u_r , u_θ and u_φ . The no-slip and impermeability conditions are imposed on the boundaries $u_\varphi(r = r_{1,2}) = \Omega_{1,2}(t)r_{1,2}\sin\theta$, $u_r(r = r_{1,2}) = 0$, $u_\theta(r = r_{1,2}) = 0$, where subscripts 1 and 2 correspond to the inner and outer spheres, respectively. Because of $\Omega_2 = 0$ in this work the subscript “1” will be omitted both for inner sphere rotation velocity Ω_1 and Reynolds number Re_1 . Before the acceleration onset and after it has stopped the inner sphere rotation velocity is constant and equal to Ω_i and Ω_f , respectively; during the acceleration it varies in accordance with the linear law $\Omega = \Omega_i + a(t - t_0)$, $t - t_0 < (\Omega_f - \Omega_i)/a$, here, a is the inner sphere acceleration. As initial conditions the stationary flow at the Re_i was taken without introducing any disturbances.

The computational algorithm is presented in detail in [10]; it is based on a conservative difference scheme for discretizing the Navier–Stokes equations in space and a semi-implicit third-order Runge–Kutta scheme for integrating in time. The discretization in space was performed using grids nonuniform in the radial and meridional directions (maximum-to-minimum cell dimension ratio in each direction varied from 1 to 4). The calculations were carried out for the dimensional parameters corresponding to the experimental conditions in [7,9], namely, for sphere radii $r_1 = 0.075$ m and $r_2 = 0.150$ m and the fluid kinematic viscosity in the layer $\nu = 5 \times 10^{-5}$ m²/s.

In our investigations the complete system of Navier–Stokes equations and reduced system with conditions of symmetry about the axis of rotation and the equatorial plane were solved. The latter system describes a two-dimensional flow which under nonstationary boundary conditions is time-dependent. The linear stability analysis of these unsteady flow was carried out, as in [5], in the quasi-stationary approximation, in which the flowfield stability is determined for a current moments of time.

3. Axisymmetric flow structure under the action of acceleration

The accelerated rotation of the inner sphere leads to a situation in which the flow is no longer symmetric about the axis of rotation even before the stability limit has been reached, though it still remains symmetric about the equatorial plane. This was experimentally established both in the case of acceleration with alternating signs (oscillations relative the axis of rotation with zero mean velocity) [11] and in the case of an acceleration constant in the value and the sign [7]. At the same time, the experiment shows that in both cases the “asymmetry” about the axis of rotation is chiefly concentrated

in a very narrow region near the equatorial plane, while outside of this zone the flow is axisymmetric. Precisely this makes it possible to investigate the effect of the acceleration and the initial Reynolds number Re_i on the variation in the flow structure in the process of acceleration and after its termination in the axisymmetric formulation. In presenting the calculated results we use the following dimensionless parameters: the dimensionless time $\tau = t/(r_1^2/\nu)$ and the dimensionless acceleration of the inner sphere $dRe/d\tau = (Re_f - Re_i)(t_f - t_i)^{-1}(r_1^2/\nu)$, here Re_f is the Reynolds number corresponding to the termination of acceleration).

In Figure 1 the angular velocity of the inner sphere Ω_1 and the ratio E_ψ/E_ϕ are presented. Before the acceleration onset all the quantities have the values corresponding to the stationary flow with Re_i and after the acceleration termination they tend to the stationary values corresponding to Re_f . Before the acceleration termination it monotonically increases. The acceleration being stopped, after a lapse of time the ratio E_ψ/E_ϕ reaches a maximum, which means a more rapid attainment of the stationary value by the meridional component of the kinetic energy, as compared with the azimuthal component. The ratio E_ψ/E_ϕ changes in a narrow time interval, not greater than 0.1 of the acceleration duration.

It is well known that under stationary boundary conditions the intensity of the meridional circulation increases with increase in the relative layer thickness δ , whereas the wavenumber of the secondary flow at the stability limit reduces: thus, in the $\delta = 1$ layer a flow with $m = 4$ is attained, while in the layer with $\delta = 1.27$ it is the flow with $m = 3$, which is confirmed by both the experiment [12] and by direct calculations [6,10]. The same is true for the flow at the end of acceleration action: the intensity of the meridional circulation increases, the greater the acceleration the greater this increase. Because of this, in the experiment an increase in the acceleration higher than certain values leads to a reduction in the secondary flow wavenumber. Thus, precisely the variation in the flow structure in the form of spatial-temporal redistribution of the circulation intensity in the meridional plane is the main cause of the reduction in the secondary flow wavenumber under the action of the acceleration.

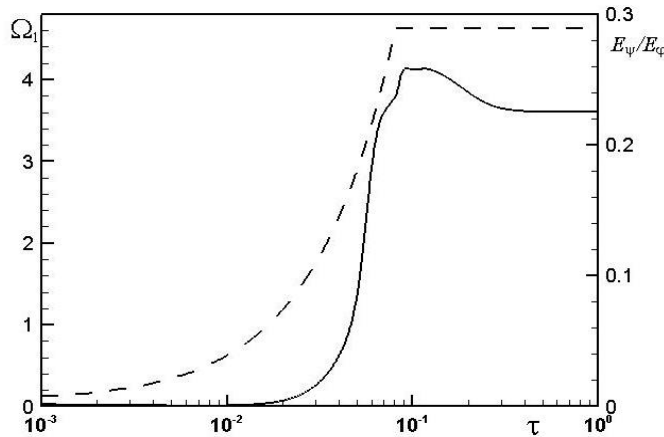


Figure 1. Flow in the process of acceleration for $Re_i = 10$, $Re_f = 520$, and $dRe/d\tau = 6328$; dashed line – angular velocity of the inner sphere; solid line – the ratio E_ψ/E_ϕ ;

4. Secondary nonaxisymmetric flows

The nonlinear calculations were performed for $Re_i = 10, 200, 450$, and 480 and $Re_f = 520$. Both finite accelerations $dRe/d\tau$ ranging from 6 to 6×10^7 and the infinite acceleration in the form of a stepwise velocity change were considered. Both in the calculations and in earlier experiments in the process of acceleration the flow remains symmetric about the equatorial plane but loses the symmetry about the axis of rotation. Depending on the acceleration, transition from the axisymmetric to the nonaxisymmetric flow occurs either during the acceleration or after its termination. The absence of the axial symmetry of the flow is particularly clearly expressed near the equatorial plane Figure 2. Precisely in this region the meridional flow velocity is maximum and the nonuniform rotation of the inner sphere leads to the formation of wave structures in the radial direction.

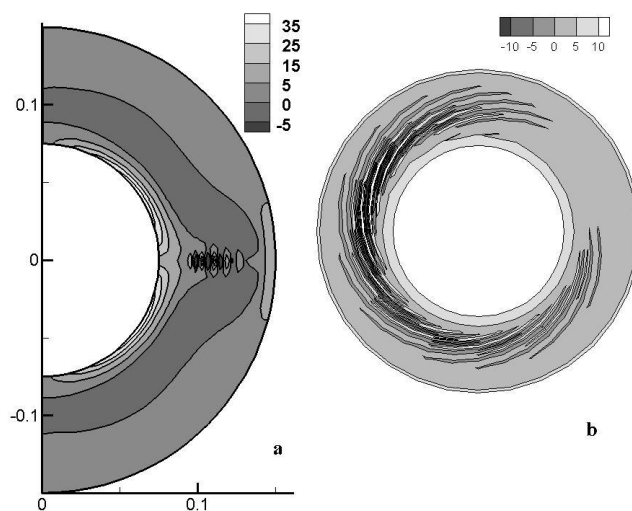


Figure 2. Meridional vorticity component in the meridional (a) and equatorial (b) planes of the flow during the acceleration.

With further development of the flow the nonuniformity in the radial direction vanishes but an asymmetry about the axis of rotation with an amplitude of about 10^{-8} is retained in the equatorial plane. Depending on the acceleration and Re_i the wavenumber in this structure can vary from 1 to 4. If an $m < 3$ mode is established, then with time it successively increases up to 3 or 4. If a structure with the wavenumber equal to 3 or 4 is formed at once, then the corresponding mode is developed in the secondary flow. In the velocity spectrum there appear peaks on the frequencies corresponding to the $m = 3$ and 4 modes. The amplitudes of these peaks increase with time, one of them attaining a constant value, whereas the other starts to decrease. The similar process of simultaneous growth of two competing modes was previously observed in the experiment [7].

Figure 3 represent the summary of calculations results of the secondary flows. For fixed values of Re_i equal to 200 and 450 an increase in the acceleration leads, as in the experiments [7,9], to the change of the four-vortex flow pattern to the three-vortex regime. In certain cases, (for example, for $Re_i = 450$) this flow pattern is conserved at arbitrarily large accelerations. For small $Re_i = 10$ the point of the flow pattern change could not be determined: despite very small acceleration values, the three-vortex secondary flow is formed. With increase in Re_i from 200 to 450 the acceleration needed for the change of the flow pattern increases by an order. The same effect of a strong Re_i -dependence of the critical acceleration was previously observed in the experiment [7].

As was shown by present calculations the dependence of the secondary flow wavenumber on the acceleration may be more complicated. This can be clearly seen in Figure 3 for $Re_i = 200$: with increase in the acceleration on the $6.3 \leq dRe/d\tau \leq 63.0$ interval the first transition from $m = 4$ to $m = 3$ is observable. Then the $m = 4$ pattern is restored on the $6.3 \times 10^3 \leq dRe/d\tau \leq 2.5 \times 10^6$ range. So on an acceleration range wider than in the experiments the secondary flow pattern can vary two times and, correspondingly, two critical values of the acceleration exist. Thus, the calculated results exhibit the same tendency in the dependence of the wavenumber on Re_i and on $dRe/d\tau$. Both with increase in the acceleration and with decrease in Re_i the wavenumber first reduces from 4 to 3 and then an inverse change of the wavenumber can occur on certain Re_i and $dRe/d\tau$ ranges.

Obtained results make it possible to explain certain experimentally observed features. In the case of infinite acceleration, the change of the flow pattern with variation in Re_i physically means that the selection of the secondary flow wavenumber is very sensitive to this quantity. The noise inevitable in experiment (deviation of the rotation velocity of the sphere from given values) and its amplification with increase in the acceleration lead to a situation in which in the vicinity of Re_i corresponding to the

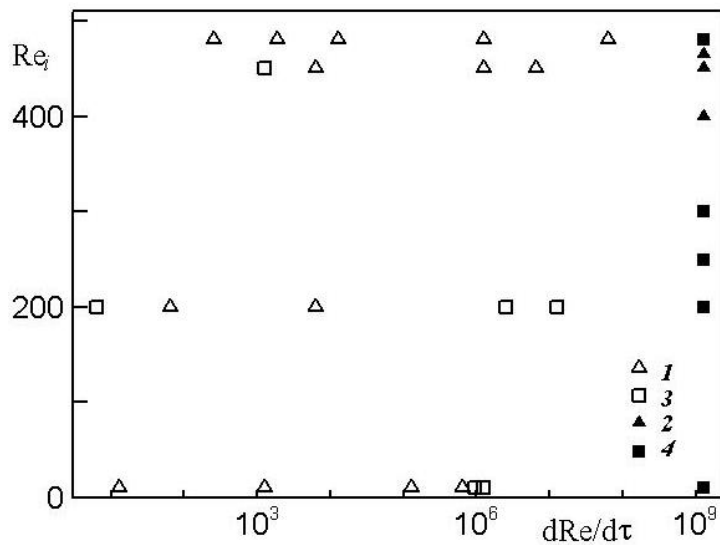


Figure 3. Dependence of the secondary flow wavenumber on Re_i and $dRe/d\tau$; (1) three-vortex flow; (2) four-vortex flow; (3, 4) relate to stepwise variation in the velocity (shown conditionally for the acceleration value of 10^9).

change of the flow pattern the wavenumber selection will occur in a random fashion, since the flow will “forget” the exact value of Re_i under the noise influence. Thus, it may be inferred that the experimentally observed indefiniteness domain is formed as a result of the joint influence of two factors, namely, the noise and the presence of a boundary of the wavenumber change with variation in Re_i . In the experiment [7] only one indefiniteness domain was observed, while in the calculations two domains were found to exist between the domains of existence of different wavenumbers. Thus, it can be inferred that one more indefiniteness domain can also exist in the experiment, on the range or higher Re_i . If this domain will be experimentally found in the future, this will serve as an additional verification of the calculated results.

5. Summary

Unsteady nonequilibrium flows of a viscous incompressible fluid in a spherical layer formed both during inner sphere rotation at a constant acceleration and after its termination are calculated. It is established that the meridional component of the kinetic energy reaches a stationary value more rapidly than the azimuthal component, which can be considered as a short-duration increase in the relative thickness of the layer. It is inferred that under the action of acceleration, periodic in value and direction, the flow in the spherical layer can be destabilized.

A direct calculation of nonaxisymmetric secondary flows in the spherical layer formed after the termination of acceleration is carried out for the first time. The calculated dependence of the secondary flow wavenumber on both the acceleration and the initial Reynolds number turns out to be more complicated than that previously obtained in the experiments. Thus, for certain values of the acceleration and the initial Reynolds number one more domain of the $m = 4$ mode existence was found. In this case, the domain of the $m = 3$ mode existence separates two domains of the $m = 4$ mode existence. The possibility of the experimental verification of this result is shown and an explanation of the experimentally observed indefiniteness domains with random selection of the secondary flow wavenumber is presented.

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